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OPTIMUM ADAPTIVE ARRAY PROCESSOR, (U)
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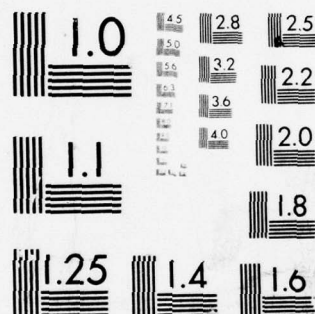
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6 OPTIMUM ADAPTIVE ARRAY PROCESSOR

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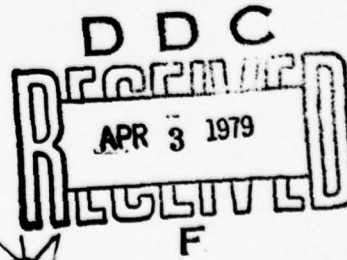
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Abstract:

This paper deals with the design and analysis of an adaptive array processor for use in passive detection of directional stochastic signals such as are found in sonar. The processor consists of a set of tapped delay-line filters, one for each array element. The algorithm used for adjusting the tap gains is a modification of the stochastic approximation method of Robbins and Monro, and it utilizes knowledge of the signal autocorrelation function and spatial direction of the target. It is shown that the final form approached by the processor is that of a space-time filter optimized in the direction corresponding to assumed target location.

The system performance is analyzed by considering a noise field consisting of a spatially isotropic component and a single directional component referred to as an interference. It is shown that although the system initially cannot discriminate between the target and the interference, it eventually acts to eliminate the effect of the interference almost completely. It is also shown how the useful signal to noise ratio increases during adaptation.

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I. Problem Statement and Objectives

The problem considered in this paper is the passive detection of a noise-like signal waveform generated by a source located in a known direction from the receiver. Typical applications of this general problem can be found in sonar detection, seismic detection and radio communications. The sonar application is the one that primarily motivates this study, and examples will be taken from the sonar area. In order to take advantage of the known directivity of the target signal, a directional receiver in the form of an array is employed to distinguish signal from noise. In the sonar application, the receiver consists of an array of hydrophones, together with an appropriate processor. Generally speaking, the processor consists of individual filters on each sensor output, a summer, a post-summation filter, a square-law device and an averaging filter. The output of the averaging filter is used to indicate the presence of a target signal.

In the absence of a target signal, the averaging filter output is the result of noise waveforms picked up by the array elements. The noise is partly far-field noise and partly locally generated. The far-field noise is often assumed to be directionally isotropic; however, there may also be directional noise sources. These directional noise sources are referred to as interference sources while the directionally isotropic component is referred to as ambient noise. In the absence of interference noise, detection of a target signal can be based simply on the presence of a directional component in the received signal. However, if interference sources can be expected to be present in the noise field, then it is necessary to define the target signal in some way to distinguish it from the directional noise components.

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This paper is concerned with developing a system for processing the outputs of a passive array of hydrophones under the following assumptions:

- (1) Target, interferences and ambient noise are assumed to be Gaussian random processes.
- (2) The sum of interferences, ambient noise, and local noise, are regarded as the effective noise, which is assumed to be statistically independent of the target signal.
- (3) The target-signal component $s_i(t)$ observed at the output of the i^{th} hydrophone is a linear time-invariant transformation of $d(t)$, the target-signal component observed at the output of an ideal isotropic hydrophone located at the origin of the coordinates. The target direction is known, together with its autocorrelation function (but not necessarily its power level).
- (4) The statistics of the noise field are completely unknown. Interferences may be present, but this is not known. If they are present, their directions are unknown.
- (5) The wavefronts of target and interferences are regarded as plane over the dimensions of the receiving array.

The proposed system consists of an adaptive linear multichannel tapped-delay-line filter and algorithms for iterative adjustment of the filter coefficients on the tapped-delay lines. The adjustment algorithm is based on the method of stochastic approximation, and it utilizes the knowledge about target direction and autocorrelation function that is assumed to be available. Thus a space-time filter optimum in a predetermined direction, and designed to suppress interference signals from other directions is produced. By varying the azimuth for which the filter is optimized the system produces a bearing response pattern which can be examined by an

operator to determine whether a target is present. In many practical situations, narrow peaks might be interpreted as targets, while broader peaks might be classified as interference.

II. Array Systems

The receiving array is assumed to consist of K omnidirectional receiving elements (sensors). Each element has a total output

$$x_i(t) = s_i(t) + n_i(t) \quad (1)$$

consisting of a signal component $s_i(t)$ plus an undesired noise component $n_i(t)$.

Consider a general array configuration consisting of individual filter $H_i(\omega)$ on each sensor output $x_i(t)$, $i = 1, 2, \dots, K$, a post-summation filter $G(\omega)$, a square-law device, and an averaging filter $h_{av}(\omega)$. The output of $h_{av}(\omega)$ is used to detect the presence of a target signal in noisy environment. See Figure 1.

Let $\phi_d(\omega)$ be the signal power spectral density, $\underline{a}(\omega)$ be the steering vector with elements $\exp(j\omega\tau_i)$, $i = 1, \dots, K$, and let $\underline{\phi}_{xx}(\omega)$ be the spectral matrix of the input. It has been shown by many authors^{1,2,3,4} that the optimum filters are given by

$$\begin{aligned} H_{op}(\omega) &= [\underline{\phi}_{xx}^*(\omega)]^{-1} \underline{a}^*(\omega) \\ G(\omega) &= \phi_d^{1/2}(\omega) \end{aligned} \quad (2)$$

for Gaussian processes and small input signal-to-noise ratios. The whole system forms a likelihood ratio detector and at the same time the output signal-to-noise ratio is maximized.

III. Adaptive Delay Line Filters

In the proposed system the individual filters $H_i(\omega)$ are constructed in the form of tapped-delay-line filters consisting of a tapped delay line, adjustable gains whose input signals are the signals at the delay-line taps, a summer to add the weighted signals, and machinery to automatically adjust the gains. The impulse response of such a discrete system is completely controlled by the weight settings.

Referring to Figure 2, let c_{ik} be the weight at the k^{th} tap on the i^{th} filter, and t_k the time delay at the k^{th} tap. Let the delayed signal be

$$\eta_{ik} = \xi_{ik} + v_{ik} = x_i(t - t_k)$$

where ξ_{ik} is the signal component and v_{ik} the noise component. Define

$$\underline{w}^T = [c_{10} c_{11} \dots c_{1M} c_{20} \dots c_{2M} \dots c_{K0} \dots c_{KM}] \quad (3)$$

$$\underline{\eta}^T = [\eta_{10} \eta_{11} \dots \eta_{1M} \eta_{20} \dots \eta_{2M} \dots \eta_{K0} \dots \eta_{KM}]$$

The mean-squared error (mse) between the desired or target signal $d(t)$ and the summer output $z(t)$, is minimized by setting the weights to

$$\underline{w}_{\text{op}} = (\underline{R}_{\xi} + \underline{R}_v)^{-1} \underline{R}_{d\xi} = \underline{R}_{\eta}^{-1} \underline{R}_{d\xi} \quad (4)$$

where \underline{R}_{ξ} and \underline{R}_v are respectively the delayed signal and noise correlation matrices of dimension $K(M+1) \times K(M+1)$. $\underline{R}_{d\xi}$ is the correlation function vector between the desired signal and the various delayed signals.

The stochastic approximation algorithm used to adjust the filter weights takes the form

$$\underline{w}_{j+1} = \underline{w}_j + 2\gamma_j \underline{R}_{d\xi} - 2\gamma_j z_j \underline{\eta}_j \quad (5)$$

In Eq. (5), the γ 's satisfy $\gamma_j = \frac{\gamma}{j^\alpha}$ with $\frac{1}{2} < \alpha \leq 1$ and $\underline{R}_{d\xi}$ is completely specified once correlation function and angular direction of the signal

are known. A block diagram of the adaptive mechanism is shown in Figure 3.

The convergence properties of algorithm (5) have been studied in a previous paper⁵ and more extensively in Ref. 6. It has been shown that algorithm (5) converges in mean square and in probability as long as the second order statistics of the input processes are bounded and if \underline{W}_{op} of Eq.(4) is a constant vector. The rate of convergence depends on the input statistics, various system parameters, and training environment. The mean-squared error is found to decrease approximately as the ^{inverse} first power of the adaptation time. The rate of convergence is essentially independent of the number of weights to be adjusted as the algorithm allows simultaneous adjustments. The size of the error, however, depends on the total number of taps and the starting point.

In order to determine the performance of the adaptive receiver, the mean values of the weights during the adaptation period are derived here for reference.

Since the summer output in Figure 2 is

$$z_j = \underline{w}_j^T \underline{\eta}_j \quad (6)$$

we have

$$\underline{w}_{j+1} = (1 - 2\gamma_j \underline{\eta}_j \underline{\eta}_j^T) \underline{w}_j + 2\gamma_j \underline{R}_d \underline{\xi} \quad (7)$$

Taking the mathematical expectations on both sides of Eq.(7), and diagonalizing the input correlation matrix \underline{R}_η such that

$$\underline{R}_\eta = \underline{P}^{-1} \underline{\Lambda} \underline{P} \quad (8)$$

we obtain

$$E[\underline{w}_{j+1}] = (1 - 2\gamma_j \underline{P}^{-1} \underline{\Lambda} \underline{P}) E[\underline{w}_j] + 2\gamma_j \underline{R}_d \underline{\xi} \quad (9)$$

where \underline{P} is an orthonormal matrix and $\underline{\Lambda}$ is the corresponding eigenvalue matrix.

Let us define a new weight vector

$$\underline{w}' = \underline{P} \underline{w}, \quad (10)$$

and a new delayed input vector

$$\underline{\eta}' = \underline{P} \underline{\eta}. \quad (11)$$

Since $\underline{R}_{d\xi} = \underline{R}_{\eta\text{-op}}$, as seen from Eq. (4), we can transform Eq. (9) into

$$E[\underline{w}'_{j+1}] = (1 - 2\gamma_j \underline{\Lambda}) E[\underline{w}'_j] + 2\gamma_j \underline{\Lambda} \underline{w}'_{\text{op}} \quad (12)$$

or

$$E[\underline{w}'_{j+1}] - \underline{w}'_{\text{op}} = (1 - 2\gamma_j \underline{\Lambda}) (E[\underline{w}'_j] - \underline{w}'_{\text{op}}) \quad (13)$$

Now consider a particular component of \underline{w}' , and for clarity no subscript or superscript denoting the component is used. Then we obtain a difference equation for

$$\begin{aligned} E[w'_j] &= \bar{w}'_j \\ \bar{w}'_{j+1} - w'_{\text{op}} &= (1 - 2\gamma_j \lambda) (\bar{w}'_j - w'_{\text{op}}) \end{aligned} \quad (14)$$

whose solution is

$$\bar{w}'_{j+1} = (\bar{w}'_1 - w'_{\text{op}}) \prod_{k=1}^j (1 - 2\gamma_k \lambda) + w'_{\text{op}} \quad (15)$$

If we choose

$$\gamma_j = \frac{1}{2(j+1)} \quad (16)$$

[This means that γ_j is the j^{th} element of a diagonal matrix having $K(M+1)$ elements], and note that

$$\prod_{k=1}^j (1 - 2\gamma_k \lambda) = \prod_{k=1}^j \left(1 - \frac{1}{k+1}\right) = \frac{1}{j+1} \quad (17)$$

we have

$$\bar{w}'_{j+1} = \frac{1}{j+1} w'_1 + \frac{j}{j+1} w'_{\text{op}} \quad (18)$$

Although the choice of γ_j given by (18) results in the simplest expression for \bar{w}'_{j+1} , other choices give similar results. See Ref. 6.

Some Remarks on the Operations of the Proposed System

(a) Choice of the Initial Weights

Although the adjustable weights can be set to any values at the beginning of the adaptive process, it is desirable to set them not too far from their optimum by using whatever information is available concerning the statistics of the noise field. The formula for calculating the optimum gains can be utilized to start the initial computation with inaccurate noise statistics. This kind of choice will shorten the adaptation period and thus reduce the cost of operation. In cases where absolutely no such information is known, the gains associated with the input delays τ_i ($i = 1, 2, \dots, K$) are set to 1 and the rest to zero. This results in a conventional power detector. As the adaptation proceeds, the whole system will gradually be transformed into an optimum one.

(b) Problem of Signal Suppression

In adaptive detection systems such as those proposed by Glaser⁷, or Jackowatz⁸, where adaptation depends on signal information produced by the processor, there is a critical input signal-to-noise ratio below which the system rejects the signal. This does not take place in our system since the signal information utilized by the algorithm is that supplied by $R_{d\hat{\xi}}$, and this is independent of input SNR. The adaptation process basically produces a space-time filter that is optimized in a given direction regardless of whether a signal is actually present in this direction or not.

(c) Unknown Signal Power

Although it may be reasonable to assume knowledge of the shape of the signal spectrum, it would generally be unrealistic to assume anything

to be known about the power level. It is shown in Appendix D of Ref. 6 that if the assumed signal power differs from the actual value by a multiplicative constant then the filter weights converge to a value multiplied by that same constant. Since a constant multiplier applied to a filter leaves the filter unchanged (except if its output is compared to some fixed threshold) lack of knowledge of the signal power level causes no real change in the basic system.

IV. Performance of the Adaptive Receiver

Our major concern is to deal with the passive detection of a sonar target in the presence of ambient noise as well as interferences from other targets.

In order to illustrate the essential procedures, we make the following simplifying assumptions:

- a) The array is linear with elements spaced a constant distance d apart.
- b) The input spectra are identical in shape over a frequency range $(0, \omega_0)$; above the frequency ω_0 the signal power is zero.
- c) The array elements are sufficiently far apart so that the ambient noise is statistically independent from element to element.
- d) There is only a single interference source.

The mean value of the detector output in the presence of the target signal is easily found to be

$$\langle y \rangle_{S+N} = \frac{1}{2\pi} \int_{-\infty}^{\infty} |G|^2 \underline{H}^T \underline{\Phi}_{xx} \underline{H}^* d\omega \quad (19)$$

where the explicit dependence of the integrand on ω is omitted for simplicity. A similar expression is obtained for $\langle y \rangle_N$, the mean value of the output in the absence of target signal, by replacing $\underline{\Phi}_{xx}$ by $\underline{\Phi}_{nn}$.

Hence the d.c. change of the output due to the presence of the target is

$$\bar{y}_{dc} = \langle y \rangle_{S+N} - \langle y \rangle_N = \frac{1}{2\pi} \int_{-\infty}^{\infty} |G|^2 \underline{H}_{ss}^T \underline{\phi}_{ss} H^* d\omega \quad (20)$$

For small input SNR the output variance can be shown to be approximately

$$\sigma_y^2 = \frac{1}{\pi T_{av}} \int_{-\infty}^{\infty} |G|^4 \{ \underline{H}_{nn}^T \underline{\phi}_{nn} H^* \}^2 d\omega \quad (21)$$

where T_{av} is the averaging time. The output SNR is then defined by

$$\bar{y}_{dc} / \sigma_y.$$

Let θ_T, θ_I , and θ be respectively the target and interference bearing and the array steering angle. These angles are measured relative to a line perpendicular to the array axis.

$$\text{Let } \tau_o = \frac{d}{c} \sin \theta_T \quad (22)$$

$$\rho_o = \frac{d}{c} \sin \theta_I \quad (23)$$

$$\hat{\tau}_o = \frac{d}{c} \sin \theta \quad (24)$$

where d is the hydrophone spacing and c is the sound velocity. θ is the independent variable in the directivity pattern $\bar{y}_{S+N}(\theta)$. Then the signal and interference delays at the i^{th} array element are

$$\tau_i = (K - i) \tau_o \quad (25)$$

$$\rho_i = (K - i) \rho_o \quad (26)$$

and the steering delay is

$$\tau_i = (K - i) \hat{\tau}_o \quad (27)$$

Let the interference delay vector be given by

$$\underline{b}^T(\omega) = [e^{j\omega\rho_1} \ e^{j\omega\rho_2} \ \dots \ e^{j\omega\rho_K}] \quad (28)$$

Then because of assumptions (c) and (d) at the beginning of this section

$$\underline{\phi}_{nn}(\omega) = \underline{\phi}_n(\omega) \underline{U} + \underline{\phi}_I(\omega) \underline{b} \underline{b}^{*T} \quad (29)$$

where \underline{U} is the identity matrix and where $\phi_n(\omega)$ and $\phi_I(\omega)$ are the ambient and interference noise spectral densities respectively. Also

$$\underline{\phi}_{xx}(\omega) = \underline{\phi}_{ss}(\omega) + \underline{\phi}_{nn}(\omega) = \underline{\phi}_d(\omega) \underline{a} \underline{a}^{*T} + \underline{\phi}_{nn}(\omega) \quad (30)$$

Note now that if optimum expressions for \underline{H} are substituted in Eqs. (19), (20), and (21) the scalar spectral functions $\phi_d(\omega)$, $\phi_n(\omega)$, and $\phi_I(\omega)$ appear only as ratios $\phi_d(\omega)/\phi_n(\omega)$, $\phi_I(\omega)/\phi_n(\omega)$ etc. Thus the assumption that the spectral shapes of signal, noise, and interference are identical permits us to replace these ratios by constant ratios S/N , I/N , etc. Also, since the signal power is assumed to vanish for frequencies beyond ω_0 , the integrals in (19), (20) and (21) are evaluated between the limits $-\omega_0$ to ω_0 . The effect of assumption (b) is therefore the same as if signal, ambient noise, and interference spectra were assumed to have flat spectra of height $S/2$, $N/2$ and $I/2$ respectively in the frequency range $[-\omega_0, \omega_0]$, and to be zero otherwise. This assumption is the one that is actually used below.

Suppose now that the transfer function of the i^{th} filter is

$$H_i(\omega) = \sum_{k=0}^M c_{ik} e^{-j\omega \Delta_{ik}} \quad (31)$$

Then Eqs. (19), (20), and (21) become:

$$\langle y \rangle_{S+N} = \frac{S}{4\pi} \sum_{i=1}^K \sum_{h=1}^K \sum_{k=0}^M \sum_{\ell=0}^M c_{ik} c_{h\ell} \int_{-\omega_0}^{\omega_0} d\omega \phi_{x_i x_h} e^{j\omega(\Delta_{h\ell} - \Delta_{ik})} \quad (32)$$

$$\overline{y}_{d.c.} = \frac{S}{8\pi} \sum_{i=1}^K \sum_{h=1}^K \sum_{k=0}^M \sum_{\ell=0}^M c_{ik} c_{h\ell} \int_{-\omega_0}^{\omega_0} d\omega e^{j\omega(\tau_i - \tau_n)} e^{j\omega(\Delta_{h\ell} - \Delta_{ik})} \quad (33)$$

and

$$\sigma_y^2 = \sum_{i=1}^K \sum_{i'=1}^K \sum_{h=1}^K \sum_{h'=1}^K \sum_{k=0}^M \sum_{k'=0}^M \sum_{\ell=0}^M \sum_{\ell'=0}^M c_{ik} c_{i'k'} c_{h\ell} c_{h'\ell'} \cdot \frac{S^2}{4\pi T_{av}} \int_{-\omega_0}^{\omega_0} d\omega \theta_{n_i n_h} \theta_{n_{i'} n_{h'}} e^{j\omega(\Delta_{h\ell} + \Delta_{h'\ell'} - \Delta_{ik} - \Delta_{i'k'})} \quad (34)$$

In these expressions $\theta_{x_i x_h}$ (or $\theta_{n_i n_h}$) is the ij^{th} element of the input (or noise spectral matrix and Eq.(34) is valid for the case of small signal-to-noise ratios.

(a) Initial Behavior

Assuming the worst case where absolutely no information about the noise field is known, the gains associated with the input delayed by τ_i ($i = 1, 2, \dots, K$) are set to 1 and the rest to zero so that a square-law detector is used at the start. Hence the output of each array element is delayed to provide maximum response in the signal direction, i.e.,

$$\underline{H}_i(\omega) = \frac{1}{N} \underline{a}^* \quad (35)$$

The weights and spacings are simply

$$\begin{aligned} c_{ik} &= \delta_{ik}/N \\ \Delta_{ik} &= \tau_i \delta_{ik} \end{aligned} \quad (36)$$

where δ_{ik} is the Kroneker delta.

Substituting Eq.(36) into Eq.(33) gives the d.c. change of the output

$$\overline{y}_{\text{d.c.}} \Big|_{j=1} = \frac{K^2 \omega_o^2}{\pi} \frac{S^2}{N^2} \quad (37)$$

where we have assumed that the integral of $e^{j\omega\Delta} \approx 0$ for $\Delta \neq 0$. Similarly, the output variance is

$$\begin{aligned} \sigma_y^2 \Big|_{j=1} &= \frac{K^2 \omega_o^2}{T_{av} \pi} \left(\frac{S}{N} \right)^2 \left[1 + \frac{I}{N} \left[2 + \frac{4}{K} \sum_{i=1}^{K-1} \frac{\sin \omega_o i \rho_o}{\omega_o i \rho_o} (K-i) \right] \right. \\ &\quad + \frac{I^2}{N^2} \left\{ 1 + \frac{4}{K} \sum_{i=1}^{K-1} \frac{\sin \omega_o i \rho_o}{\omega_o i \rho_o} (K-1) \right. \\ &\quad \left. \left. + \frac{2}{K^2} \sum_{i=1}^{K-1} \sum_{h=1}^{K-1} \left[\frac{\sin \omega_o (i-h) \rho_o}{\omega_o (i-h) \rho_o} + \frac{\sin \omega_o (i+h) \rho_o}{\omega_o (i+h) \rho_o} \right] (K-i)(K-h) \right\} \right] \end{aligned} \quad (38)$$

Dividing Eq.(37) by the square root of Eq.(38) gives the output signal-to-noise ratio, which becomes approximately [10]

$$\text{SNR}_1 \approx \frac{1}{2} \left(\frac{T_{av} \omega_o}{\pi} \right)^{1/2} \frac{S}{N} K \left\{ 1 + 2 \frac{I}{N} + \frac{I^2}{N^2} \left(\frac{2}{3} K + \frac{1}{3K} \right) \right\}^{-1/2} \quad (39)$$

if the maximum frequency processed is very high such that $\omega_o \rho_o \gg 1$ and the terms associated with ρ_o make negligible contributions except for $i = h$.

For most cases of practical interest, the number of hydrophones in an array is large, $K \gg 1$, so that for ambient-noise-dominated environment

$$\text{SNR}_1 \propto K \left(\frac{S}{N} \right) \text{ when } \left(\frac{N}{I} \right)^2 \gg \frac{2}{3} K \quad (40)$$

and for interference-dominated environment

$$\text{SNR}_1 \propto K^{1/2} \left(\frac{S}{I} \right) \text{ when } \left(\frac{N}{I} \right)^2 < \frac{2}{3} K \quad (41)$$

The average output of the squarer, \bar{y} , yields the so-called directivity pattern, which may be obtained by varying the electrical time delays.

In the signal direction $\hat{\tau}_o = \tau_o$ and for $\omega_o \rho_o \gg 1$ Eq.(32) becomes:

$$\bar{y}_1(\theta = \theta_T) \approx \frac{\omega_o}{\pi} K \frac{S}{N} \left[1 + \frac{I}{N} + K \frac{S}{N} \right] \quad (42)$$

In the interference direction $\hat{\tau}_o = \rho_o$, hence

$$\bar{y}_1(\theta = \theta_I) \approx \frac{\omega_o}{\pi} K \frac{S}{N} \left[\left(1 + \frac{S}{N} \right) + K \frac{I}{N} \right] \quad (43)$$

and in any other direction

$$\bar{y}_1(\theta) \approx \frac{\omega_o}{\pi} K \frac{S}{N} \left(1 + \frac{S}{N} + \frac{I}{N} \right) \quad (44)$$

These expressions indicate quite clearly that there are peaks in the directivity pattern in the signal and interference directions ($\theta = \theta_T$ and $\theta = \theta_I$) and that the system does not discriminate in any way against the interference.

(b) Final Behavior

1. Optimum Gains

Since the adaptive processor converges to the optimum detector, the final values of the gains are given by Eq.(4). If the input signal-to-noise ratio is small $\frac{R}{\eta} \approx \frac{R}{\nu}$; also it can be shown⁶ that if all the spectra are similar $\frac{R}{\eta} = K_1 \frac{R}{\nu}$ where $K_1 \geq 1$ is a constant. Hence we may use

$$\underline{W}_{\infty} = \underline{R}_{\nu}^{-1} \underline{R}_{d\xi} \quad (45)$$

This equation is not suitable for obtaining analytical expressions for the optimum gains, and therefore we approximate the gains by a simple Fourier integral expression:

$$c_{ik} \approx \frac{1}{2\pi} \int_{-\infty}^{\infty} H_i(\omega) e^{j\omega t_k} d\omega \quad (46)$$

where $H_i(\omega)$ is the i^{th} optimum filter given in Eq.(2) with $\frac{\phi}{xx}$ replaced by $\frac{\phi}{nn}$.

Then, using Eq.(29) in (2) we find that the final expression for the entire filter is given by

$$\underline{H}_{\infty}(\omega) = [\frac{\phi}{nn}]^{-1} \phi_d^{1/2} \underline{a}^* = \frac{\sqrt{S}}{N} [\underline{U} - \frac{\underline{b}^* \underline{b}^T}{K + N/I}] \underline{a}^* \quad (47)$$

The i^{th} row of $\underline{H}_{\infty}(\omega)$ is

$$H_i(\omega) = \frac{\sqrt{S}}{N} [a_i^* - \frac{\sum_{k=1}^K b_i^* b_k^* a_k^*}{K + N/I}] \quad (48)$$

so that the impulse response is

$$h_i(t) = \frac{1}{2\pi} \frac{\sqrt{S}}{N} \int_{-\infty}^{\infty} [e^{j\omega(t-\tau_i)} - \frac{\sum_{k=1}^K e^{j\omega(t-\rho_i+\rho_k-\tau_k)}}{K + N/I}] d\omega \quad (49)$$

It is clear that the impulse response of the individual filters can be completely specified by setting the gains according to

$$c_{ik} = \frac{\sqrt{S}}{N} \left(\delta_{ik} - \frac{1}{K + N/I} \right) \quad (50)$$

at delays

$$\Delta_{ik} = \rho_i - \rho_k + \tau_k \quad (51)$$

Note that since a practical delay line can supply only discrete delays it is generally not possible to obtain the exact values required by (51). It is assumed that the delay lines have a sufficient number of taps so that the performance degradation produced by this is negligible.

ii. Signal-to-Noise Ratio

If we substitute Eq.(50) into Eqs.(33) and (34), and follow the same steps leading to Eq.(39), we obtain the signal-to-noise ratio of the detector output:

$$\begin{aligned} \text{SNR}_\infty = & \frac{1}{2} \left(\frac{T_{av} \omega_o}{\pi} \right)^{1/2} \frac{S}{N} \frac{K(K-1)}{K + N/I} \frac{N/I}{K + N/I} \\ & \left\{ 1 - \frac{4}{K(K-1 + N/I)^2} \sum_{i=1}^{K-1} (K-i) \frac{\sin \omega_o i \rho_o}{\omega_o i \rho_o} \right. \\ & + \frac{2}{K^2 (K-1 + N/I)^2} \sum_{i=1}^{K-1} \sum_{h=1}^{K-1} (K-i)(K-h) \\ & \left. \left[\frac{\sin \omega_o (i-h) \rho_o}{\omega_o (i-h) \rho_o} + \frac{\sin \omega_o (i+h) \rho_o}{\omega_o (i+h) \rho_o} \right] \right\}^{1/2} \quad (52) \end{aligned}$$

If $\omega_o \rho_o \gg 1$, then

$$\begin{aligned} \text{SNR}_\infty & \approx \frac{1}{2} \left(\frac{T_{av} \omega_o}{\pi} \right)^{1/2} \frac{S}{N} (K-1) \frac{1+N/(K-1)I}{1+N/KI} \left[1 + \frac{1}{3} \frac{(K-1)(2K-1)}{K(K-1+N/I)^2} \right]^{1/2} \\ & \geq \frac{1}{2} \left(\frac{T_{av} \omega_o}{\pi} \right)^{1/2} \frac{S}{N} (K-1) \quad (53) \end{aligned}$$

Equation (52) or (53) gives the asymptotic performance of the adaptive array processor. Since the training period is finite, the actual

signal-to-noise ratio is lower than that given by Eq.(52) or (53). As would be expected Eq.(52) is just the output signal-to-noise ratio of an optimal (likelihood ratio) detector first investigated by Schultheiss⁹ and then by Tuteur¹⁰ from a simpler formulation, but under the same assumption of similar input spectra over $(0, \omega_0)$.

iii. Directivity Pattern

Although the directivity pattern can be obtained by using Eq. (50) in (32), the fact that (50) represents essentially the optimum filter makes it possible to use Eq.(47) in (19) to put the result into a more compact matrix notation:

$$\bar{y}_\infty(\theta) = \frac{1}{2\pi} \frac{S}{N} \int_{-\omega_0}^{\omega_0} d\omega \hat{\underline{a}}^{*T} \left[\underline{U} - \frac{\underline{b} \underline{b}^{*T}}{K+N/I} \right] \left[\underline{U} + \frac{I}{N} \underline{b} \underline{b}^{*T} + \frac{S}{N} \underline{a} \underline{a}^{*T} \right] \left[\underline{U} - \frac{\underline{b} \underline{b}^{*T}}{K+N/I} \right] \hat{\underline{a}} \quad (54)$$

where it is to be understood that the \underline{a} 's and \underline{b} 's are functions of ω .

Expansion of the integrand results in twelve terms typified by the term $\hat{\underline{a}}^{*T} \underline{b} \underline{b}^{*T} \underline{b} \underline{b}^{*T} \hat{\underline{a}} (K + N/I)^{-2}$. The vector product that remains after removing $\underline{b} \underline{b}^{*T} \underline{b} = K$ is

$$\sum_{i=1}^K \sum_{h=1}^K e^{j\omega(\hat{\tau}_h - \rho_h + \rho_i - \hat{\tau}_i)}$$

We consider the integral of the summand to be approximately zero except for $i = h$, in which case it is $2\omega_0$. Thus the contribution of this term is $2K^2\omega_0$. The other eleven terms are evaluated similarly except that in combination such as $\hat{\underline{a}}^{*T} \underline{a} \underline{a}^{*T} \hat{\underline{a}}$ or $\hat{\underline{a}}^{*T} \underline{b} \underline{b}^{*T} \hat{\underline{a}}$ one must consider also whether the independent variable $\hat{\tau}_0$ is near τ_0 or near ρ_0 respectively. Thus, if $\hat{\tau}_0 = \tau_0$, $\hat{\underline{a}}^{*T} \underline{a} \underline{a}^{*T} \hat{\underline{a}} = K^2$, if $\hat{\tau}_0$ differs from τ_0 the contribution of $\hat{\underline{a}}^{*T} \underline{a} \underline{a}^{*T} \hat{\underline{a}}$ is only K .

By utilizing considerations of this sort we obtain the following approximate expressions for the directivity pattern:

a) For angles far away from target or interference

$$\bar{y}_{\infty}(\theta) \approx \frac{\omega_o}{\pi} \frac{S}{N} \left\{ K-1 + \left(\frac{N/I}{K+N/I} \right)^2 + \frac{KN/I}{(K+N/I)^2} + \frac{S}{N} \left[K-1 + \left(\frac{N/I}{K+N/I} \right)^2 \right] \right\} \quad (55)$$

In the signal direction ($\theta = \theta_T$)

$$\bar{y}_{\infty}(\theta = \theta_T) \approx \frac{\omega_o}{\pi} \frac{S}{N} \left\{ K-1 + \left(\frac{N/I}{K+N/I} \right)^2 + \frac{KN/I}{(K+N/I)^2} + \frac{S}{N} K \left[K - \frac{2K}{N/I+K} + \frac{2K-1}{(K+N/I)^2} \right] \right\} \quad (56)$$

and in the interference direction ($\theta = \theta_I$)

$$\bar{y}_{\infty}(\theta = \theta_I) \approx \frac{\omega_o}{\pi} \frac{S}{N} \left\{ K \left(\frac{N/I}{K+N/I} \right)^2 + \frac{K^2 N/I}{(K+N/I)^2} + \frac{S}{N} K \left(\frac{N/I}{K+N/I} \right)^2 \right\} \quad (57)$$

Note that if the noise is ambient-dominated $N/I \gg K$, and in this case these three expressions are essentially the same as Eqs.(42), (43), and (44). This is to be expected since in the absence of interference, and if all spectra have the same shape the simple filter of Eq.(35) is optimum. On the other hand, if the interference noise is large $N/I \ll K$, the three equations above become respectively:

$$\bar{y}_{\infty}(\theta) \approx \frac{\omega_o}{\pi} \frac{S}{N} (K-1) \left(\frac{S}{N} + 1 \right) \quad (55a)$$

$$\bar{y}_{\infty}(\theta = \theta_T) \approx \frac{\omega_o}{\pi} \frac{S}{N} (K-1) \left(K \frac{S}{N} + 1 \right) \quad (56a)$$

$$\bar{y}_{\infty}(\theta = \theta_I) \approx \frac{\omega_o}{\pi} \frac{S}{N} K \left[\left(\frac{N/I}{K} \right)^2 \left(\frac{S}{N} + 1 \right) + \frac{(N/I)^2}{K} \right] \rightarrow 0 \quad (57a)$$

Thus the optimum filter tends to suppress the interference noise in all directions, and for large I/N it produces a dip in the response pattern in the interference direction.

(d) Adaptive Behavior

During the adaptation period the transfer function of the i^{th} filter is given by Eq.(31). The c 's appearing in this equation are represented by the vector \underline{w} of Eq.(3), which changes with time according to Eq.(5). The variation of the elements of \underline{w} during the training period therefore determines

the adaptive behavior of the processor. Since in Eq.(5) z_j and y_j are random, the \underline{w}_j are also random. Thus we consider only the expected values of \underline{w}_j . According to Eq.(18) the expected values of \underline{w}_j is related to the initial and final value by the general formula

$$E[\underline{w}_{j+1}] = p_j \underline{w}_1 + q_j \underline{w}_\infty \quad (58)$$

where \underline{w}_1 is the initial value of \underline{w} and \underline{w}_∞ the final value, and where the p_j and q_j are functions of j that depend on the choice of the weighting sequence γ_j of the stochastic approximation algorithm. It is clear that $p_1 = 1$, $q_1 = 0$, $p_\infty = 0$, $q_\infty = 1$.

Combining Eqs.(31) and (58) we can write

$$\underline{H}_{j+1} = p_j \underline{H}_1(\omega) + q_j \underline{H}_\infty(\omega) \quad (59)$$

where $\underline{H}_1(\omega)$ and $\underline{H}_\infty(\omega)$ denote respectively the initial and final forms of the transfer function vectors. For the particular example considered here, $\underline{H}_1(\omega)$ is given by Eq.(35) and $\underline{H}_\infty(\omega)$ by Eq. (2).

Basically, we are required to evaluate the following three integrals

$$A_{j+1} = \frac{1}{2\pi} \int_0^{\omega_0} |G|^2 (p_j \underline{H}_1 + q_j \underline{H}_\infty)^T \underline{\Phi}_{ss} (p_j \underline{H}_1 + q_j \underline{H}_\infty)^* d\omega \quad (60)$$

$$B_{j+1} = \frac{1}{\pi T_{av}} \int_{-\infty}^{\infty} |G|^4 \{ (p_j \underline{H}_1 + q_j \underline{H}_\infty)^T \underline{\Phi}_{nn} (p_j \underline{H}_1 + q_j \underline{H}_\infty)^* \}^2 d\omega \quad (61)$$

$$C_{j+1} = \frac{1}{2\pi} \int_0^{\omega_0} |G|^2 (p_j \hat{\underline{H}}_1 + q_j \hat{\underline{H}}_\infty)^T \underline{\Phi}_{xx} (p_j \hat{\underline{H}}_1 + q_j \hat{\underline{H}}_\infty) d\omega \quad (62)$$

where in Eq.(61) the \underline{H} 's use the steering vector $\hat{\underline{a}}$ rather than the signal delays \underline{a} .

Note that in general

$$\begin{aligned} (p_j \underline{H}_1 + q_j \underline{H}_\infty)^T \underline{\Phi} (p_j \underline{H}_1 + q_j \underline{H}_\infty)^* &= p_j^2 \underline{H}_1^T \underline{\Phi} \underline{H}_1^* + q_j^2 \underline{H}_\infty^T \underline{\Phi} \underline{H}_\infty^* \\ &+ 2p_j q_j \underline{H}_1^T \underline{\Phi} \underline{H}_\infty^* \end{aligned} \quad (63)$$

The first two terms of Eq.(63) are already available from the previous two sections on the initial and final behaviors. Thus, if we set $A_{j+1} = \sum_{i=1}^3 A_{j+1}^{(i)}$ (1)

$$A_{j+1}^{(1)} = p_j^2 \frac{1}{2\pi} \int_{-\infty}^{\infty} |G|^2 \underline{H}_1^T \underline{\Phi}_{ss} \underline{H}_1^* d\omega = p_j^2 \bar{y}_{d.c.} \Big|_{j=1} \quad (64)$$

where $\bar{y}_{d.c.} \Big|_{j=1} = \frac{K^2 \omega_o}{\pi} \frac{S^2}{N^2}$ by Eq.(37). Similarly

$$A_{j+1}^{(2)} = q_j^2 \frac{1}{2\pi} \int_{-\infty}^{\infty} |G|^2 \underline{H}_{\infty}^T \underline{\Phi}_{ss} \underline{H}_{\infty}^* d\omega = q_j^2 \bar{y}_{d.c.} \Big|_{j=\infty} \quad (65)$$

$$\begin{aligned} A_{j+1}^{(3)} &= 2p_j q_j \frac{1}{2\pi} \int_{-\infty}^{\infty} |G|^2 \underline{H}_1^T \underline{\Phi}_{ss} \underline{H}_{\infty}^* d\omega \\ &\approx 2p_j q_j \frac{K^2 \omega_o}{\pi} \left(\frac{S}{N}\right)^2 \frac{K-1 + N/I}{K + N/I} \end{aligned} \quad (66)$$

Similar expressions can be obtained for B_{j+1} and C_{j+1} ; B_{j+1} requires the evaluation of six terms, C_{j+1} the evaluation of three terms. For details see [6]. Then the output signal-to-noise ratio is

$$SNR_{j+1} = \frac{A_{j+1}}{(B_{j+1})^{1/2}} \quad (67)$$

and the directivity pattern is

$$\bar{y}_{j+1}(\theta) = C_{j+1} \quad (68)$$

Eqs. (65) and (66) have been computed by means of a digital computer, using the optimum γ_j given by Eq.(16). Figure 4 shows the variation of output signal-to-noise ratio. Figure 5 shows the variation in the directivity pattern for target and interference well separated in bearing. The way in which the interference is suppressed during the adaptation is clearly evident in Figure 5.

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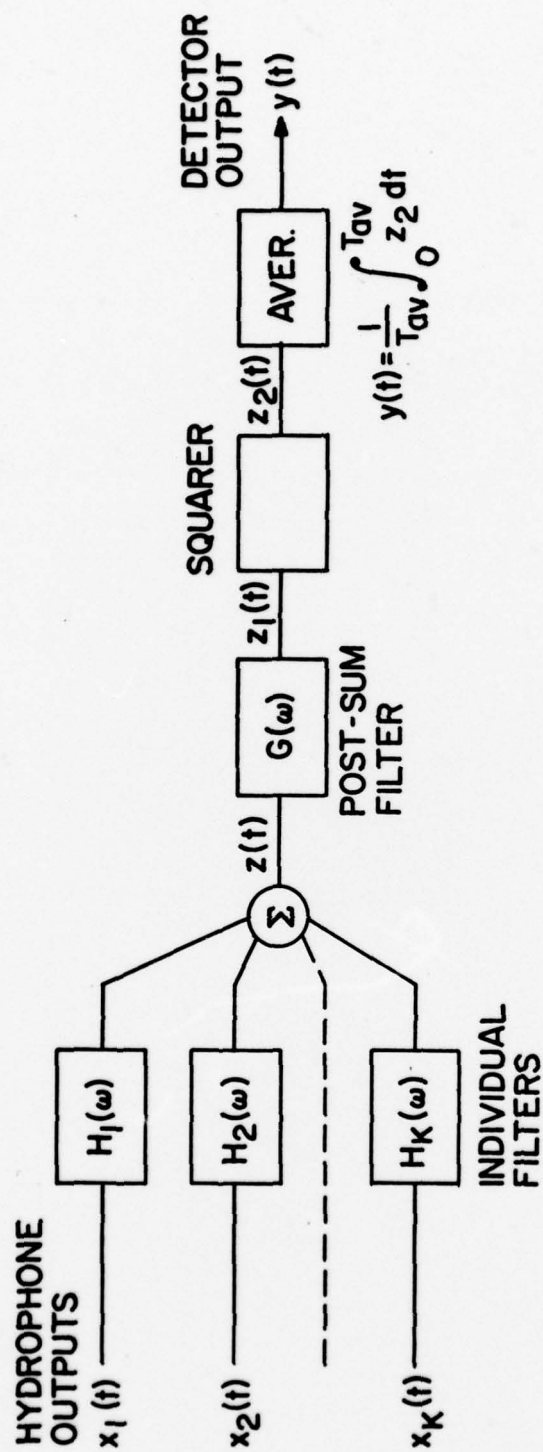


Figure 1. General Array Processor

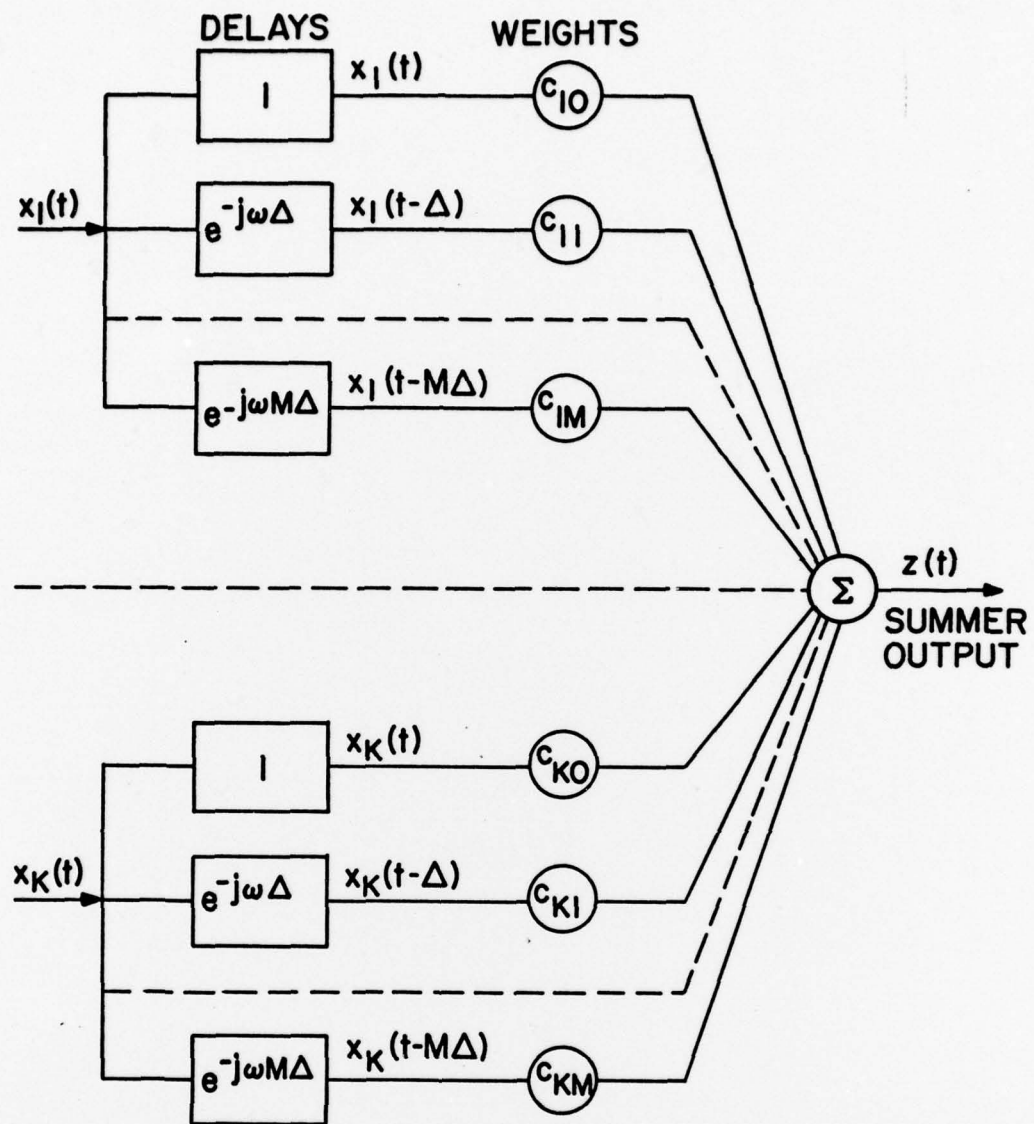


Figure 2. Tapped-Delay-Line Filter in an Array

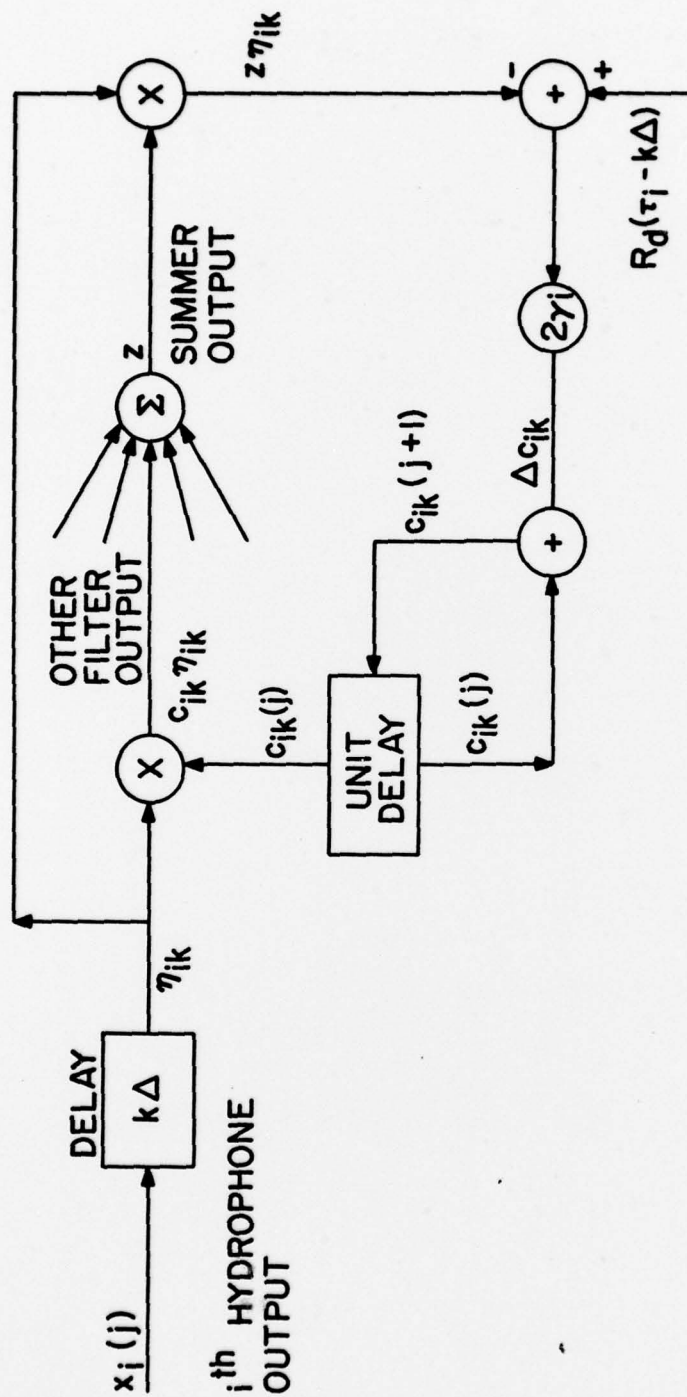


Figure 3. Adaptive Mechanism

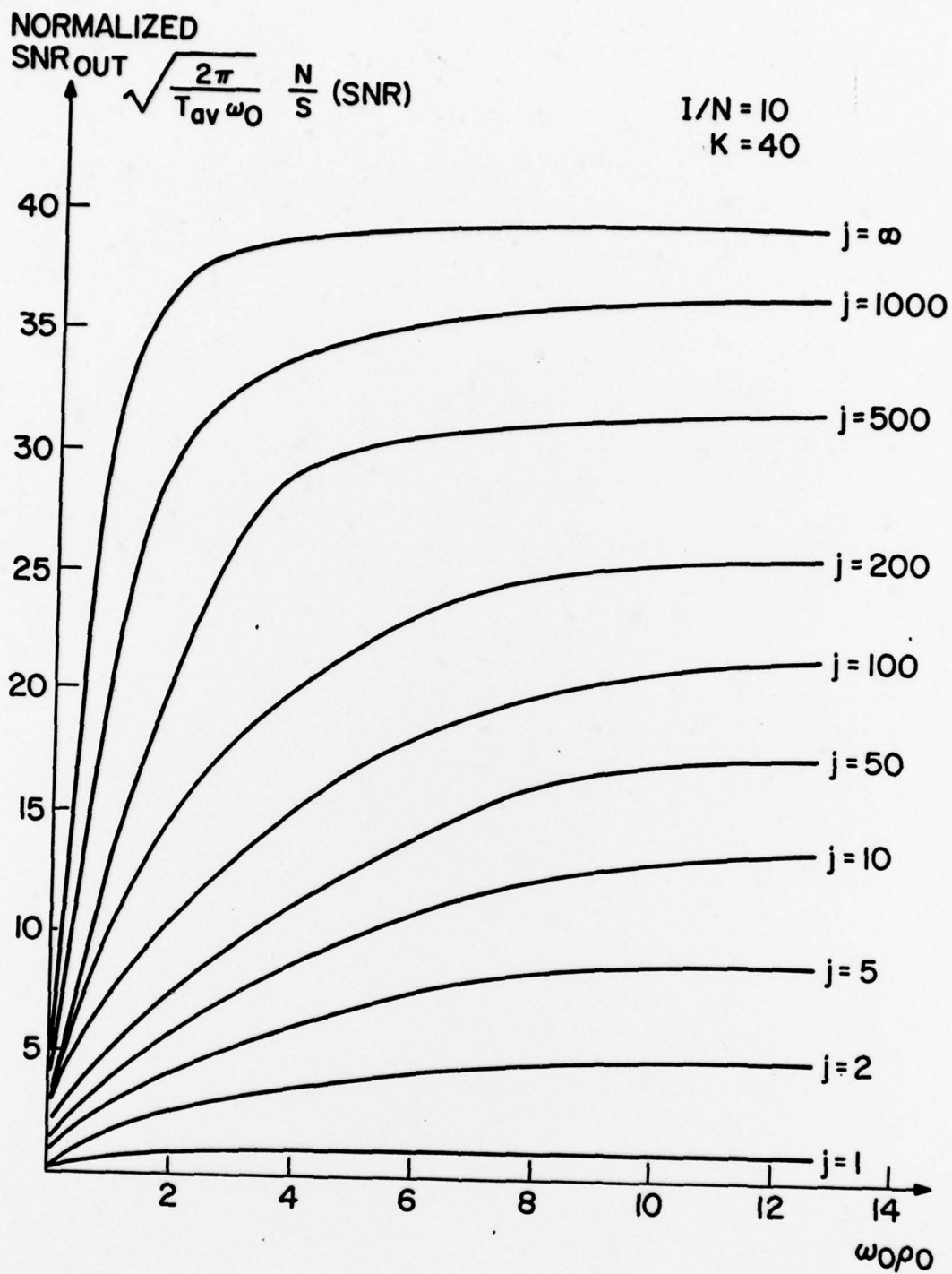


Figure 4. Variation of Output SNR

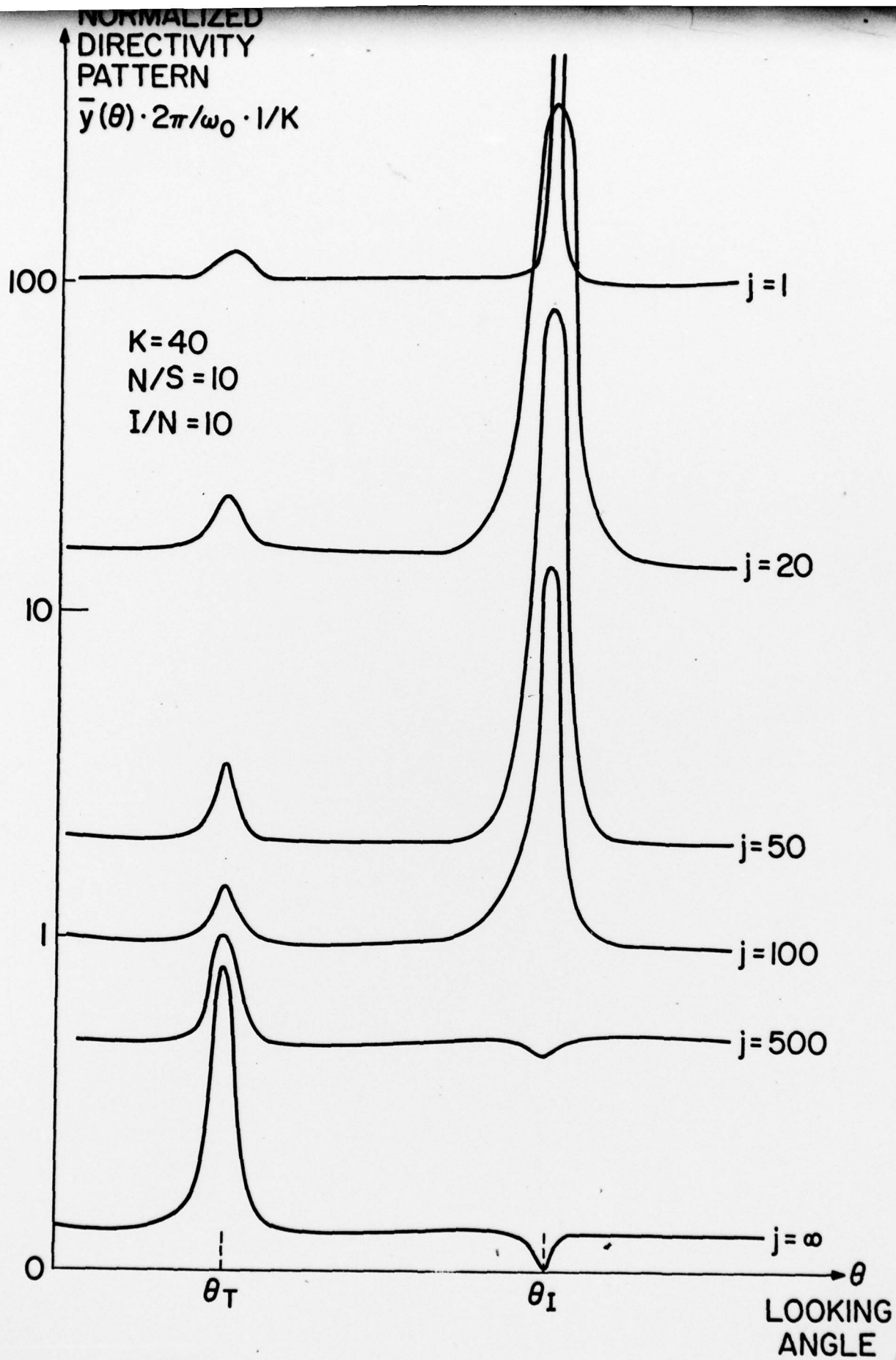


Figure 5. Variation of Directivity Pattern